A Price Theoretic Model of Search Intermediation by Online Platforms

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Abstract

We analyze the incentives of online search intermediaries to reduce the consumers' search costs when the intermediary imperfectly observes consumers' preferences. In our model, the intermediary manipulates the demand in two downstream product markets by choosing which product is the search default, and the cost of finding the alternative. Subsequently the available supply in each market is allocated in a perfect price equilibrium. The decentralized search decisions mimic those of a social planner with the same search technology, and thus a welfare-maximizing intermediary would set zero search costs. By contrast, an intermediary who maximizes seller revenue will optimally maintain positive search costs so that the default can be used to steer consumers to the market where they generate the most revenue.

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1 Introduction

Many of today's leading companies can be thought of as multi-sided platforms.[1] The main service such platforms provide in order to attract their customers is facilitating value creating transactions between buyers and sellers of goods or services. In markets with almost homogeneous goods (e.g. Uber), the platform centralizes the matching of buyers to sellers; but in many differentiated product markets (e.g. Airbnb, Amazon, Upwork, Kayak), the platform presents options and lets the buyer pick. The platform controls two aspects of the buyer experience: the search default and the search costs. For example, a platform can feature one brand by putting it on top of the search results list; design the web page that makes it hard for users to find alternatives without additional clicks; provide or avoid the interface for head-to-head comparision of competing products; and subtler things like invest in backend technologies for faster navigation of the web site; not restrain the individual sellers' obfuscation policies[2]. Even though search intermediation is a large part of the modern economy, the economic implications of the intermediaries' decisions have not been fully explored in the literature.

In this paper we analyze how a platform's choice of default product and search costs affects consumer surplus, seller revenue, or overall welfare and study the optimal platform's policies to maximize these objectives. We find that minimizing search frictions is best when maximizing welfare or consumer surplus, though with upward sloping supply this does not achieve the first best consumer surplus. By contrast, positive search costs are required to maximize seller revenue, since in the presence of search costs, defaults can be used to steer consumers to the high-priced or price-inelastic market. Furthermore, randomized defaults may be necessary. Since most search intermediaries are paid by sellers, either on a per transaction or revenue-sharing basis, one might reasonably expect platforms to create search frictions and strategically manipulate defaults in order to raise revenues.

The following example will illustrate the crucial idea that absent transfers, the second-best solution for an intermediary is to have positive search costs. Imagine a search aggregator for air travel, such as Kayak or Momondo, whose business model is to facilitate search for cheap air tickets and take fees from the airlines and travel agencies. For simplicity, there are two downstream companies, United and American, that have similar flight routes, so that consumers preferences between the two are idiosyncratic (brand loyalty, airline benefit card, etc.). How should the intermediary structure the search process in terms of a) search frictions and b) featured product, to increase its revenue? On the one hand, low search frictions allow consumers find the airline from which they derive the most utility, in terms of better match and better price. Assuming prices are set close to marginal costs, frictional search also generates the most value. On the other hand, low search costs do not allow the platform to capture much of the generated value. By maintaining search frictions, the platform sacrifices a part of overall welfare but instead obtains an option of steering searchers to the airline that generates more revenue for the platform. If the loss in total welfare is smaller than the gain in extracted surplus, the platform has no incentives to optimize search.

We study the platform's incentives to create search frictions and manipulate default prod-

[1] Hagiu and Altman (2017).

[2] Ellison and Wolitzky (2012).

ucts using a simple price-theoretic model. There are two perfectly competitive downstream markets, A and B; each produces a single good. The consumers are located on a Hotelling segment with consumers on the far right preferring good A and consumers on the far left preferring good B and consumers in the middle being indifferent between the two goods. Upon entering the platform, a consumer is presented with a *default good* that he can either purchase or search for the other good. The platform controls which good is the default. The choice of default good can be deterministic or stochastic. The platform also controls the cost of finding the alternative, or a *search cost*. The search cost captures all tangible and intangible costs related to search, such as the consumer's limited attention span, spending time and effort. In this environment, the intermediary has two ways of controlling the user search experience. First, they can invest in technology to lower the search cost, e.g. develop a more efficient web page. Second, they can change the default search result. Its incentive for manipulating these controls depends on whether its objective is aligned with users or sellers in the downstream market.

We start our analysis in Section 3 with the first-best scenario in which the platform manually allocates consumers into the downstream markets. We establish that in this case the platform would create two non-overlapping consumer segments with one of them assigned to market A and the other to market B. The size of optimal consumer segments depend on the platform's objective, and we consider maximizing consumer surplus, joint seller profits, total welfare and joint sellers' revenue. The objectives of maximizing the consumer surplus and the sellers' profits are in conflict. Specifically, suppose market B is cheaper and has more elastic supply. Then starting from the welfare-maximizing allocation, shifting more consumers to market B increases the consumer surplus and decreases the sellers' profits, while shifting more consumers to market A does the complete opposite (Theorem 2). Although the first-best allocations could potentially be implemented using a rich set of tools, such as transfers and search subsidies, in many practical situations the platform has only limited tools to organize the search process.

In Section 5, we explore what economic outcomes the platform can achieve by manipulating the search cost and default good. First, we establish that the first-best welfare is achievable in a frictionless market equilibrium. Second, no other first-best objective can be achieved in a market equilibrium, and so optimizing the platform's revenue, sellers' profits or consumer surplus generally requires frictional search and manipulating the default product (Theorem 6). In the rest of the paper, we study the optimal platform's policies. We start with the benchmark case when the prices in downstream markets are fixed, and then move on to the main case of endogenous prices.

In the benchmark case when the prices are fixed in the downstream markets, consumer search decisions do not affect prices in the second stage of the model, and so there is only one direct channel of how search costs affect surplus on both sides of the market. First, we find that when the platform maximizes consumer surplus net of search costs (net consumer surplus), the optimal search cost is zero. Indeed, the platform prefers that the buyers incur the minimal cost and find the best match. Both goals are accomplished by setting zero search costs. On the contrary, positive search costs are necessary for the intermediary to maximize the total revenue. Suppose good A has a higher price than good B. The revenue-maximizing platform would prefer for each consumer to buy good A rather than good B, though buying good B is better than them not buying at all. To push traffic to market A, the platform can make good A the default search result, but this only has impact if search costs are positive.

When the downstream markets have limited supply, the analysis is more subtle. Steering buyer traffic to one of the markets by increasing the search cost affects the prices which feeds back into the searchers' incentives to switch into this market in the first place. Therefore there is an additional indirect effect of search costs on surplus on both sides of the market. Again, suppose good A has a higher price than good B, and suppose the platform strives to maximize the sellers' profits. Directing buyers into market A leads to higher price in market A and lower price in market B. If the supply of good B is inelastic, price for good B drops too much. The resulting loss in revenue on inframarginal consumers in market B can outweigh the gain in price on marginal consumers shifted into market A.

In our first main result, we show that maximizing the sellers' profits requires positive search costs and defaulting consumers into the market with smaller price semi-elasticity of supply. Specifically, suppose that good A has semi-*in*elastic supply, which means that the supply of good A is inelastic or good A is expensive, and good B has semi-elastic supply, which means that its supply is elastic or it is cheap. Defaulting consumers into market A and raising the search cost above zero increases the sellers' profits (Theorem 9).

Now suppose the platform's objective is maximizing the consumer surplus net of search costs (net consumer surplus, NCS). Again, let the supply of good A be less semi-elastic relative to the supply of good B. We show that the first-best allocation of consumers between the markets requires that more consumers be in market B relative to the market equilibrium with zero search costs. Indeed, when more consumers are moved into market B, the price of good B barely changes while the price in market A drops, that results in lower average transaction price. To implement the feat of moving more consumers in market B in market equilibrium, consider a policy that defaults consumers into market B and raises the search costs. This kind of steering policy worked in case of optimizing the sellers' profits.

The second main result of the paper shows that, surprisingly, the net consumer surplus cannot be increased by increasing search costs (Theorem 10). Even though steering consumers into market B increases the *gross* consumer surplus by inducing more favorable prices for inframarginal consumers, the increase in search costs is always greater and offsets the gain in price.

We conclude that since most search intermediaries are paid by sellers, either on a per-transaction or revenue-sharing basis, platforms have strong incentives to maintain or create search frictions and to manipulate defaults strategically. Absent of search costs, the market generates the largest total surplus for the participants on the platform. However, maximizing revenue requires steering consumers toward the products that would bring the most revenue. To do so, the platform features these products and then creates hurdles for the consumers to find alternative options. Therefore, the platform is optimally willing to hamper consumers search process.

Suppose the platform maximizes its revenue which is a fixed share $\tau > 0$ of the sellers' revenue. The platform's incentives are similar to the case with optimizing the sellers' profits but with more emphasis on price. We show that the platform optimally sets positive search costs, and defaults consumers with greater probability to the market with higher marginal revenue, where the marginal revenue is equal to price plus the inverse of semi-elasticity of supply (Theorem 12).

Under vertical integration of the intermediary with one of the product markets (e.g.

Amazon selling Amazon products on its marketplace, Google Flight Search on Google), the intermediary has strong incentives to give prominence to its own product. Suppose the platform earns a profit share of $\tau < 1$ in market A, but takes all the profit in market B, e.g. they own a subsidiary that does the supply in market B, with the given upward sloping marginal cost curve. Then it is optimal to set positive search cost and default consumers into market B unless A's semi-elasticity of supply is $1/\tau$ lower than the own supply.

Related Literature. Our paper is a contribution to the literature on search intermediation. Most of the literature on platform markets has focused on network effects of participation (Armstrong (2006); Rochet and Tirole (2006); Weyl (2010)), and the implications of these effects for optimal pricing, either under monopoly or oligopoly market structure. All these papers treat the interaction of users on the platform as a blackbox. Instead, we are concerned with the intermediary's incentives to organize the users' search when they are already on the platform.

The existing literature on search intermediation addressed some aspects of consumer search, such as diversion and information withholding (Armstrong *et al.* (2009); Hagiu and Jullien (2011); Ellison and Wolitzky (2012); Athey and Ellison (2011); Edelman and Lai (2016)), but either did not study the intermediary's incentives or did not study defaults in combination with direct search costs.

Ellison and Ellison (2009) provide the empirical evidence of obfuscation from a price comparison website and find that sellers intentionally hamper information to make consumers less price sensitive. Ellison and Wolitzky (2012) develop a theoretical model in which obfuscation is individually rational for oligopolistic firms. In their model, a firm "obfuscates" search by making a consumer pay more to learn the price. In our paper search costs are controlled by the platform, thus being exogenous to the sellers, and we study the intermediary's incentives to reduce them. Also, unlike in this and other standard search-theoretic models (Stahl (1989); Ellison (2005)), we assume that the consumers know the equilibrium prices and directly incur a cost of switching to an alternative good.

The most closely related paper to ours is Hagiu and Jullien (2011), who study an intermediary's incentives to divert consumer search. In their model, diversion is platform's manipulation of default product and they assume that the platform observes consumer characteristics and conditions the default on consumer type. We, on the contrary, assume that diversion is anonymous—the default good is the same for all consumers. This distinction results in opposite incentives for the platform to reduce search costs: In Hagiu and Jullien (2011), the intermediary would always want to decrease consumers' search costs if it could, while in our paper we establish the opposite.

Another closely related paper by Armstrong, Vickers and Zhou (2009) explores how manipulation of default product affects competition in search markets and implications for consumer surplus, profit, and welfare. Similar to our setting, the platform chooses a default product, but in their model, there is a range of differentiated products each produced by a separate firm. Armstrong *et al.* (2009) show that the prominent firm sets lower price because it has a large population of randomly drawn consumers (relatively price elastic), versus the other firms who also have consumers who are dissatisfied with their other options and are therefore rather more price inelastic. The market structure is different in our paper, with competitive markets for each good, that results in higher price for the prominent good as opposed to lower in Armstrong *et al.* (2009).

2 The Model of Mediated Consumer Search

This section builds a model of a two-sided market with horizontally differentiated consumers who search for products in downstream markets, and their search process is mediated by the platform. We are interested in how intermediaries can affect market outcomes by channeling consumer demand towards different products. In later sections we will use the model to analyze the intermediary's incentive in setting two parameters of search process: search costs and default product.

There are two products, and a continuum of horizontally differentiated consumer types. We focus on the case where consumers know the characteristics of the two products ex-ante, and so the cost involved in finding the non-default product should be interpreted as a search or switching cost, rather than an information acquisition cost. We assume that consumer tastes are private information, so that the intermediary cannot deliver personalized defaults or search results.

There are two types of goods available, A and B. There are a measure one of potential buyers for these good, with privately-known types x uniformly distributed on the unit interval $[0,1]$. Buyers have unit demand, and differentiated preferences over the two goods. Type x has valuation $v_A(x)$ for good A, where $v_A(x)$ is strictly increasing and continuously differentiable in x, $v_A(0) = 0$ and $v_A(1) = \bar{v}$. They have valuation $v_B(x)$ for good B, where $v_B(x)$ is strictly decreasing and continuously differentiable in x, $v_B(1) = 0$ and $v_B(0) = \bar{v}$. Preferences are thus "Hotelling", in the sense that buyers who value the A good more value the B good less. A special case arises when valuations are symmetric around $x = 0.5$ with $v_A(x) = v_B(1-x)$; we will refer to this as the case of symmetric valuations. Another (more) special case occurs when $v_A(x) = v_B(1-x) = \bar{v}x$; we will call this the case of "uniform valuations".

We assume that buyers are risk neutral and have quasi-linear utility. Buyers payoff equals to their valuation for the good they purchased, less the price paid, less any search costs (introduced below).

The supply side is modeled by upward-sloping supply curves $S_j(p)$, $j = A, B$, where $S_j(p)$ weakly increasing and continuously differentiable in p. When supply is perfectly elastic at some prices p_A and p_B, we will say we are in the "fixed-price" case, as regardless of the search technology, markets will always clear at these prices. Unless specified otherwise, in the rest of the paper the supply curves have the familiar interpretation of (short-run) marginal cost curves (where marginal costs include payments to the platform) under the assumption that supply is perfectly competitive. But even when marginal costs are constant, an upward-sloping supply curve may arise due to endogenous seller entry: higher prices may induce sellers in the broader market to affiliate with the platform, as those prices allow them to cover fixed entry costs.

Central to our paper is the idea that the platform may allocate buyers to a default market (the market for A or the market for B). Buyers who want to switch to the other market must pay a search cost. Both the default and the search cost may be manipulated by the platform.

The timing is as follows. Buyers simultaneously play a two stage game. In the first stage, each buyer is assigned a default good by the platform, privately known to them. We assume that the default good is assigned to be good A with probability α, independently across buyers. Some important special cases include the case where $\alpha = 1$ (the default is always good A) and $\alpha = 0$ (the default is always good B). Since the types are private, the platform cannot offer type-specific defaults (e.g. assigning buyers to the market for the good they value most). Each buyer then simultaneously makes the decision whether to switch their assignment to the other good, paying a switching cost of $c \geq 0$ to do so.

In the second stage, the market for each good is cleared. Each consumer now belongs to a market (either their default good or the one they switched to). Let the set of consumers in the market for good A be $\mathcal{A}$, and similarly those for B be $\mathcal{B}$. Then this induces demand curves:

$$\begin{aligned} D_A(p) &= \{x \colon x \in \mathcal{A}\ ,\ v_A(x) \geq p\} \\ D_B(p) &= \{x \colon x \in \mathcal{B}\ ,\ v_B(x) \geq p\} \end{aligned} \tag{1}$$

Each market is cleared at the price that equates supply and demand:

$$\begin{aligned} S_A(p_A) &= D_A(p_A) \\ S_B(p_B) &= D_B(p_B) \end{aligned} \tag{2}$$

To simplify later analysis, we make a "full coverage" assumption. In the usual Hotelling model, this means assuming that demand is large relative to supply (marginal costs), so that everyone ends up purchasing a good in equilibrium. Similarly, we will say that the market is fully covered if in the (unique) equilibrium with $c = 0$ (i.e. in the absence of search frictions), all types buy a product.

Assumption 1 (Full Coverage)**.** *There exists a consumer type x_0 such that $1 - x_0 \leq S_A(v_A(x_0))$ and $x_0 \leq S_B(x_0)$.*

The assumptions posits that there is excess supply in both markets at the price some type x_0 is willing to pay, which in turn ensures that every type earns positive surplus (types below x_0 can buy B, types above x_0 can buy A).

3 Platform's First-Best Solutions

In this section, we study how the platform would allocate consumers between the downstream markets if it could observe their types and force them to stay in those markets. These are the allocations the platform would implement had it had rich set of tools for steering consumers. In the main part of the paper we compare these allocations to those achievable in a more realistic setting with imprefect tools: search costs and default product. We find that, regardless of the platform's objective, the first-best solutions require sharp segmentation into the markets (in the sense made clear below), and welfare-maximizing allocation is between consumer-optimal allocation and producer-optimal.

Only for this section, assume that the stage of decetralized search is replaced with the platform manually allocating consumers to markets conditional on their type x. That is, the platform chooses sets $\mathcal{A}$ and $\mathcal{B}$ such that $\mathcal{A} \cup \mathcal{B} \subset [0,1]$. The second stage is the same, with the demands being formed according to (1) and the markets clear at perfectly competitive prices.

We are interested in understanding the platform's incentives to allocate consumers between the downstream markets under several objective functions. What is the optimal allocation of consumers when the platform maximizes the welfare? The consumer surplus? The joint seller profits? The sellers' revenue? Unless specified otherwise, we assume throught the paper that the supply curves are equal to the marginal cost curves and sellers' fixed costs are zero.

The breakdown of consumers into sets $\mathcal{A}$ and $\mathcal{B}$ can potentially be complex and the sets can overlap so that matching of consumers to the markets is "fuzzy". The next lemma shows that the fuzzy allocation is never optimal, and there are no leftover consumers.

Lemma 1. *If the platform maximizes weighted average of consumer and producer surplus, or the sellers' revenue, then there is a cutoff x^{FB} such that consumers with $x < x^{FB}$ are assigned to market B and consumers with $x > x^{FB}$ are assigned to market A.*

The optimal cutoff x^{FB} depends on the platform's objective, and we can say something about it.

Theorem 1. *The first-best welfare is attained by using cutoff x^{FBW} that satisfies:*

$$v_A(x^{FBW}) - p_A^{FBW} = v_B(x^{FBW}) - p_B^{FBW}, \tag{3}$$

where p_A^{FBW} and p_B^{FBW} are competitive prices that realize in the second stage.

There is an intuitive explanation for (3). The left hand side is the type-x^{FBW} consumer's utility of being in market A, while the right hand side is his utility of being in market B. Therefore, type x^{FBW} is indifferent between being in A and B. As a side note, monotonicity of v_A and v_B implies that at fixed prices ${p_A}^{FBW}$ and ${p_B}^{FBW}$, all consumers do not have private incentives to switch the allocation they are given by the platform. This observation will lead to Theorem 6 below.

Denote the first-best cutoffs for maximizing consumer surplus, producer surplus and sellers' revenue by x^{FBC}, x^{FBP} and x^{FBR}, respectively.

Theorem 2. *Let p_j and ε_j be price and supply elasticiy in market j evaluated at the welfare-maximizing allocation with cutoff x^{FBW}.*

- *If $p_A/\varepsilon_A > p_B/\varepsilon_B$, then $x^{FBP} < x^{FBW} < x^{FBC}$.*
- *If $p_A/\varepsilon_A < p_B/\varepsilon_B$, then $x^{FBP} > x^{FBW} > x^{FBC}$.*

Also, $x^{FBR} \neq x^{FBW}$.

The result shows that if the platform has full control, the allocations are distinct for every objective the platform might have. Moreover, the consumer and producer surplus are in conflict. Specifically, starting at the welfare-maximizing allocation, if market j is more expensive and/or has less elastic supply, then moving more consumers into market j increases producer surplus and decreases consumer surplus. And conversely, shifting consumers away from market j decreases producer surplus and increases consumer surplus.

The question is whether the first-best cutoffs can be implemented using the restricted set of platform's tools, search cost c and default product α. This is what we turn on to now.

4 Market Equilibrium

In this section we prove the existence and uniqueness of the market equilibrium with consumer search. We also give initial comparative static of how key policy variables, search costs and default product, affect the structure of the equilibrium.

The model described in Section 2 is an extensive form game of incomplete information. The solution concept we use is subgame perfect equilibrium with price equilibrium in the second stage. In the second-stage of the game, prices p_j will be determined by equating supply and demand in market j. At that point, it is weakly dominant for each type x to either buy the good (if $v_j(x) \geq p_j$) or not (if $v_j(x) < p_j$). As a result, they will earn the payoff of

$$u_j(x, p_j) = \max\{v_j(x) - p_j, 0\}.$$

Moving back to the first stage, buyers must decide whether to search given their type and their current (default) product. Since there are a continuum of buyers, their individual actions have no effect on the market-clearing price in the second stage, and so they behave as though they face fixed prices p_j in each market.

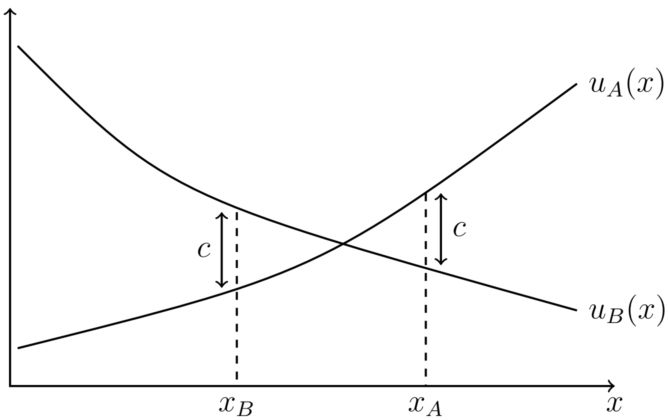
Figure 1a illustrates the trade-off in the consumer search decision. In the face of fixed prices, the payoff from participating in market $u_A(x, p_A)$ is increasing in x since valuations are rising; conversely the payoff from participating in market B, $u_B(x, p_B)$, is falling. Whenever the difference between the two exceeds the search cost c, agents who are defaulted into their less preferred market will switch.[3] As depicted, this gives rise to a pair of thresholds x_B and x_A such that types below x_B who have a default good of A will choose to search; and those with type above x_A with a default good of B will choose to search, where the thresholds are defined by:

$$\begin{aligned} u_A(x_A, p_A) - u_B(x_A, p_B) &= c \\ u_B(x_B, p_B) - u_A(x_B, p_A) &= c \end{aligned} \tag{4}$$

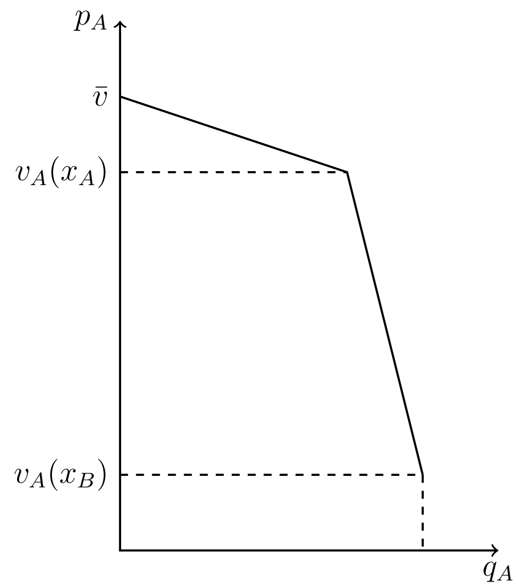
Types between $[x_B, x_A]$ do not search.

Since the search decisions take this threshold form for all possible prices p_j, they will also be pinned down by thresholds x_A and x_B in any equilibrium. Demand in each market comes from a mixture of the extremal types who are willing to search for their preferred product if necessary (for good A, types from x_A to 1), and the never-searchers between $[x_B, x_A]$ who were given the product as a default.

[3] In the special case where search costs are zero, it is possible that a positive mass of agents may be indifferent about searching versus not searching. We refine this away by assuming that indifferent types do not search.



(a) Interim payoff functions.



(b) Demand curve in the case of uniform valuations.

Figure 1

Because these two groups are ordered in their willingness-to-pay, for fixed thresholds x_B and x_A the demand curves $D_j(p, x_A, x_B)$ take a piecewise form. For example, in market A:

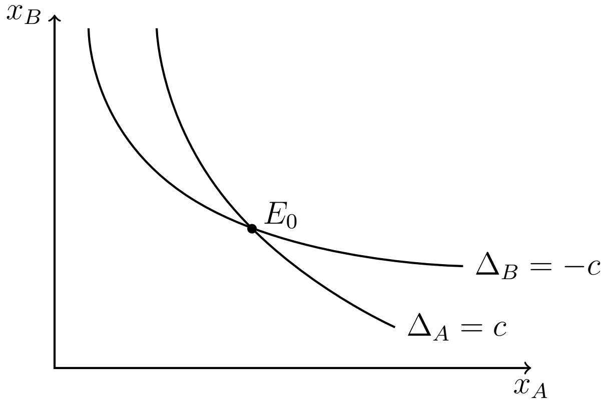
$$D_A(p, x_A, x_B) = \begin{cases} 1 - x_A + \alpha\,(x_A - x_B) & p \le v_A(x_B) \\ 1 - x_A + \alpha\left(x_A - v_A^{-1}(p)\right) & v_A(x_B) < p \le v_A(x_A) \\ 1 - v_A^{-1}(p) & v_A(x_A) < p \le \bar{v} \\ 0 & p > \bar{v} \end{cases} \tag{5}$$

Figure 1b shows the (inverse) demand curve for product A in the special case of uniform valuations, which imply piecewise linear demand. Notice its "bowed-out" shape, with the slope of the demand curve falling in prices. The intuition for this is that price changes at low prices have little effect on demand, because the inframarginal consumers are non-searchers, and only a fraction α of those will respond to price changes (the remainder stick with their default of product B). The platform can thus exert some control over the price elasticity of demand faced by sellers in each market by manipulating defaults and search costs.

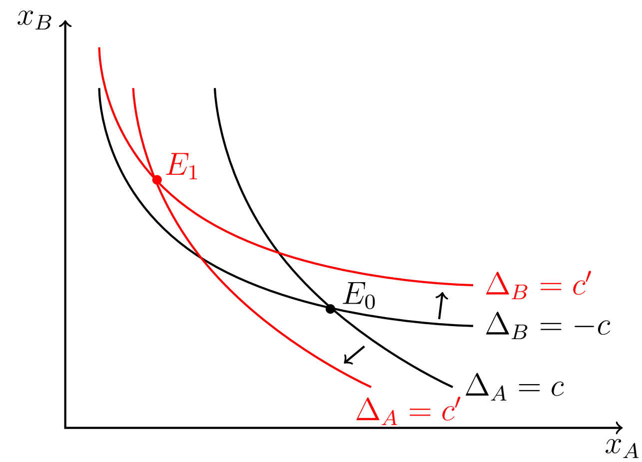
Figures 1a and Figure 1b respectively held prices and thresholds fixed. We now want to look for an equilibrium of this game: a set of prices (p_A, p_B) that equate supply and demand, where demand depends on the search thresholds (x_A, x_B); and a set of search thresholds (x_A, x_B) that are consistent with optimal search behavior when consumers correctly anticipate prices (p_A, p_B).

Theorem 3 (Existence and Uniqueness)**.** *The equilibrium exists and is unique.*

Now we give a sketch of the proof and then provide the comparative static results. To show existence, first eliminate the prices from the equilibrium analysis, by noting that the



(a) E_0 is the unique equilibrium.



(b) Comparative statics when switching cost c decreases, $c' < c$.

Figure 2: The curves $\Delta_A = c$ and $\Delta_B = -c$ are respectively the loci of types who are indifferent about searching for good A (good B) if defaulted to market B (market A).

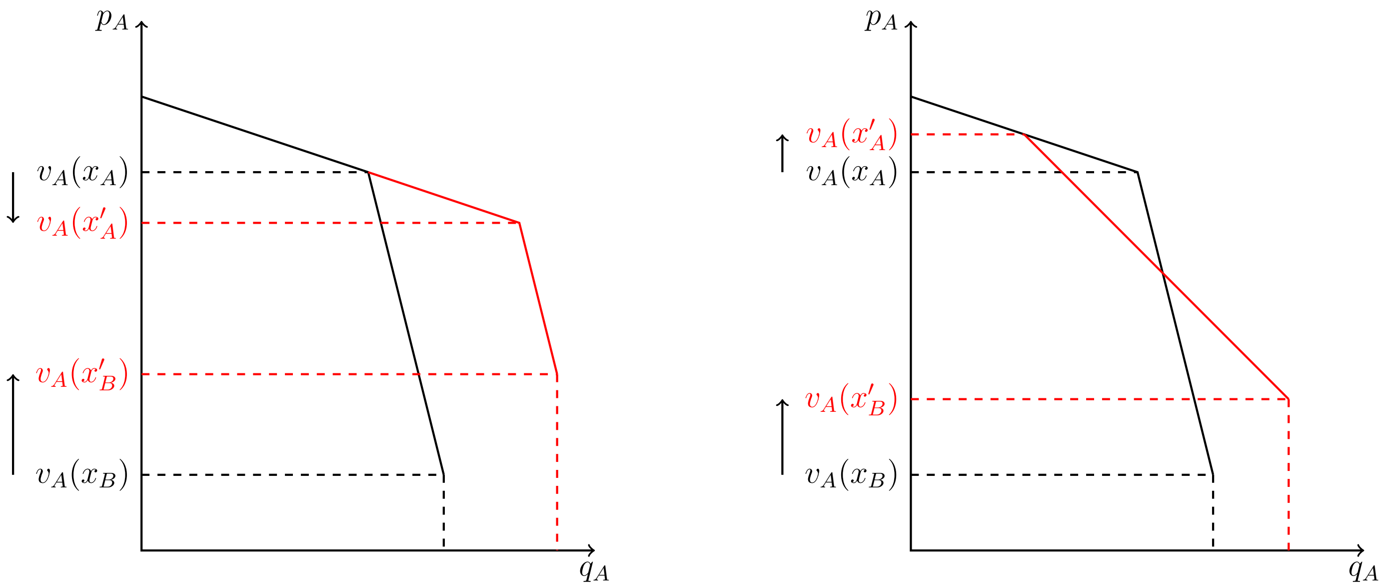
thresholds (x_A, x_B) are sufficient to pin down demand, and given exogenous supply, therefore sufficient for prices. We can write consumer utility in the first stage as $u_j(x; x_B, x_A) \equiv u_j(x; p_j(x_B, x_A))$. Let $\Delta_A(x_A, x_B) \equiv u^A(x_A; x_B, x_A) - u^B(x_A; x_B, x_A)$ and $\Delta_B(x_A, x_B) \equiv u^A(x_B; x_B, x_A) - u^B(x_B; x_B, x_A)$. The functions Δ_A and Δ_B are equal to the payoff gain from switching to market A from market B for types x_A and x_B respectively. In equilibrium, it must be that x_A is just indifferent about switching to A and paying c, and x_B is just indifferent between switching to A and being compensated with c. Thus any potential equilibrium thresholds must satisfy the following pair of equations:

$$\begin{aligned} \Delta_A(x_A, x_B) &= c \\ \Delta_B(x_A, x_B) &= -c \end{aligned} \tag{6}$$

Figure 2a plots graphical illustration of (6) in threshold space.

Curves $\Delta_A = c$ and $\Delta_B = -c$ are downward sloping: in both equations, the payoff difference from participating in market A relative to B is increasing in both x_B and x_A, both because of direct effects (e.g. in the first equation, type x_A values A more as x_A increases) and competition (whenever either x_A or x_B increases, market B gets more competitive and market A less so). Intuitively, curve $\Delta_A = c$ should be everywhere steeper than the second, as a small change in x_A has a big effect (both direct and through competition) which must be met by a big change in x_B to hold the payoff differential constant and equal to c. The complete proof is in the appendix.

We conclude this section with some comparative statics on the design parameters available to the platform: the search costs c, and the default probability α. Consider Figure 2b. When search costs decrease, the curve $\Delta_A = c$ shifts left (for the same x_B, the threshold type x_A who searches is lower), while the curve $\Delta_B = -c$ shifts up (for the same x_A, the threshold type x_B who searches is higher). In the new equilibrium E_1 we have lower x_A and higher x_B and thus more search.



(a) Demand curve when switching cost c decreases.

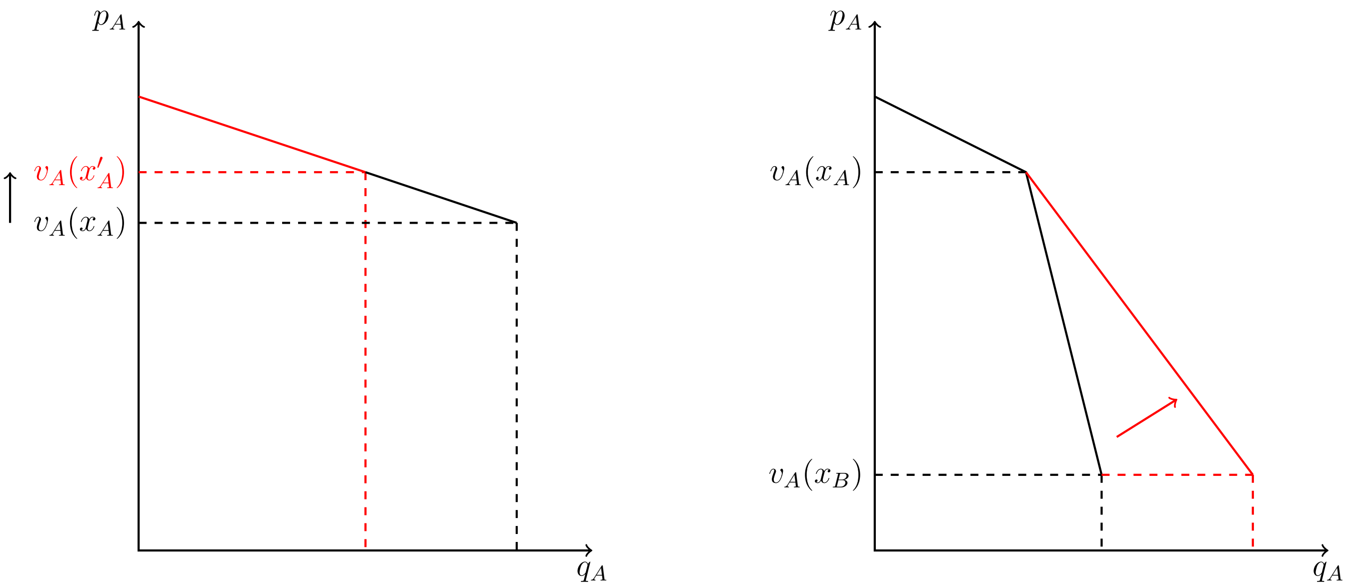
(b) Demand curve when the default assignment to good A increases.

Figure 3

Theorem 4 (Comparative statics in search costs)**.** *Equilibrium thresholds $x_B \leq x_A$ are decreasing and increasing respectively in the search cost c. As $c \to 0$, they converge to a single threshold $x_A = x_B \equiv x_0$; as $c \to \infty$, they hit the boundaries of the type-support $x_B = 0$ and $x_A = 1$. When A is the default product ($\alpha = 1$), p_A is increasing and p_B is falling in c; conversely when B is the default ($\alpha = 0$), p_B is increasing and p_A is falling.*

Decreasing the search costs thus changes the demand curves in a predictable way, as illustrated in Figure 3a. In market A, the threshold x_A falls and so the kink in demand occurs at a lower value, raising demand in the middle of the distribution. But because the threshold x_B rises, demand from low value buyers falls. The same is true of market B. The effect on prices is ambiguous, as in general it depends on the supply of goods A and B. But in the particular case where one of the goods is assigned as the default for all consumers, search costs uniformly increase demand for that good, implying that prices (weakly) rise. Thus the platform can steer demand to a particular good by making it the default and by making the platform more "opaque" (increasing search costs).

A change in α, the probability a consumer sees a default of A, potentially has ambiguous effects on the thresholds. On the one hand, there is a direct effect of increased demand for good A, since more consumers are assigned it as a default. An so, in equilibrium consumers will be less willing to search for A in anticipation of increased demand from non-searchers and higher prices, and so one might expect both x_B and x_A to rise. This need not be the case in general though, and depends on how the Δ_A and Δ_B curves change as α changes, which in turn depends on the supply curves. In the special case of perfectly elastic supply, this second channel is shut down: prices do not respond to demand shifts, and so increasing α unambiguously increases demand for A and decreases it for B.



(a) Demand change when A is the default ($\alpha = 1$) and switching cost c increases.

(b) Demand change when the default assignment to good A increases (α increases) holding thresholds fixed.

Figure 4

The next result shows that even with inelastic supply, the second channel is weaker, and both thresholds increase in α.

Theorem 5 (Comparative statics in default probability)**.** *When all buyers purchase a good, equilibrium thresholds x_B and x_A are (weakly) increasing in the default probability α.*

5 Search Costs and Default Product: Welfare Analysis

In this section we study how consumer search costs and the choice of default product affect the equilibrium consumer surplus, joint seller profits and the intermediary's revenue. To the extent the intermediary controls those variables, the analysis sheds light on the intermediary's incentives on reducing search costs and diverting consumers. We find that it is necessary to have positive search costs to maximizing the joint seller profits. We also prove the alignment theorem that states that the policies that maximize consumer surplus and welfare coincide.

First, we establish that only first-best welfare can be achieved in market equilibrium, and so maximizing platform's revenue, producer or consumer surplus requires frictional search and manipulating the default product.

We say that an allocation $(\mathcal{A}, \mathcal{B})$ of consumers between the markets is *implementable* if there are c and α such that $(\mathcal{A}, \mathcal{B})$ realizes in the market equilibrium. The *gross consumer surplus* (GCS) is the joint consumer utility they obtain ignoring the search costs, The *(net) consumer surplus* (NCS or CS) includes the incurred search costs.

Theorem 6. *The first-best welfare is attained in market equilibrium by setting $c = 0$. The first-best net consumer surplus is not attainable in market equilibrium.*

For the proof of the first part of the theorem holds, note that when $c = 0$, the thresholds x_A and x_B coincide, denote it by $x_0 = x_A = x_B$. From (6) we have that $v_A(x_0) - p_A = v_B(x_0) - p_B$. Compare this expression with (3) and obtain $x_0 = x^{FBW}$.

For the second part of the theorem, observe that the only implementable allocation under $c = 0$ is welfare-maximizing. Implementing the consumer optimal allocation requires implementing the cutoff x^{FBC}, which is distinct from x^{FBW} by Theorem 2. Getting there at least requires increasing c, and so the net consumer surplus will be below the first-best.

Similarly, implementing profit- or revenue-maximizing first-best allocations will require frictional search. Since sellers do not incur search costs, it can be possible to implement those allocations. In this paper we do not provide the full implementability result but instead characterize the second-best net consumer and producer surplus and check whether they require $c > 0$.

We start with the analysis of benchmark case when prices are fixed to lay down the basic intuition and then move on to the main case of competitive prices.

5.1 Benchmark: Fixed Prices

In this section, we consider a benchmark when the prices in both markets p_A and p_B are fixed. This is an easy case because c and α only affect the consumers in the first stage of the game when they search, while the indirect effect on prices is absent.

Theorem 7. *Suppose prices are fixed. Then consumer surplus is strictly decreasing in c for any α.*

Proof by picture: the prices are fixed, so the consumer surplus is just the area under the u_A and u_B terms in Firgure 1a, or an α-weighted average in the middle part. Clearly things get worse as c increases and the thresholds move outwards.

The next result characterizes the choice of the default and the search cost that maximizes the joint seller revenue. Since the prices are fixed, the revenue maximization effectively requires to force as many consumers as possible to buy the more expensive product.

Theorem 8. *Suppose prices are fixed. The platform maximizes revenue by choosing the more expensive product as the default, and choosing the highest c such that everyone still buys a product.*

5.2 Perfect Price Competition in Downstream Markets

This section is the heart of the paper in which we analyze the main case with upward sloping supply curves. Price p_j in market $j \in \{A, B\}$ is pinned down in price equilibrium:

$$S_j(p_j) = D_j(p_j),$$

where D_j is defined in (5) and S_j is increasing.

The next result is the first main result of the paper and establishes that frictional search is necessary to maximize the joint sellers' profits.

Theorem 9. *Let the supply curves be equal to the marginal cost curves and sellers' fixed costs be zero. A platform that wants to maximize joint seller profits should set $c > 0$ and default consumers into the less semi-elastic market, that is the market with higher p_j/ε_j.*

Suppose market A has semi-inelastic supply, i.e. a higher price and/or less elastic supply relative to market B. Then the platform can incrase joint seller profits by moving more consumers into market A. It can do so by setting good A as the default and raising the search cost c. Further, although good A as the default generates a gain in profits, the deterministic default need not be optimal. Defaulting everyone into A weakens demand in market B, and so it might be optimal to use the stochastic default.

Now we will sketch the proof of Theorem 9 because it is illustrative for the economic forces behind the result and also because other results will have similar economic intuition. The economics of the intermediary's intervention can be understood through the transfer of buyers between the two markets. Denote by q_j the quanitity sold in market j. Any change in platform's policy induces the equilibrium change in quantities (dq_A, dq_B). The seller profits in market A is

$$PS_A = \int_0^{p_A} S_A(t)dt. \tag{7}$$

from where we can find the change in total seller surplus:

$$dPS = dPS_A + dPS_B = q_A dp_A + q_B dp_B$$

In price equilibrium, $S_j(p_j) = q_j$, and the platform's policies affect only the demand. Thus,

$$dp_j = dq_j / S_j'(p_j).$$

And so,

$$\begin{aligned} dPS &= \frac{q_A}{S_A'(p_A)} dq_A + \frac{q_B}{S_B'(p_B)} dq_B \\ &= \frac{p_A}{\varepsilon_A} dq_A + \frac{p_B}{\varepsilon_B} dq_B, \end{aligned}$$

where ε_j is price elasticity of supply in market j.

When all buyers purchase a product,

$$dq_A = -dq_B.$$

Therefore,

$$dPS = \left(\frac{p_A}{\varepsilon_A} - \frac{p_B}{\varepsilon_B} \right) dq_A. \tag{8}$$

The key aspect concerns which dq_A the platform can induce when its only tools are c and α. Towards this, find the expression for dq_A via changes in thresholds x_A and x_B. Let a differential increase in search cost dc induce (dx_A, dx_B). Increase in x_A leads to mass $(1-\alpha)dx_A$ of buyers to stop searching into market A and mass αdx_B to start searching into market B. Therefore,

$$dq_A/dc = -(1-\alpha)dx_A/dc - \alpha dx_B/dc. \tag{9}$$

Start with $c = 0$. By Theorem 4, x_A increases in c while x_B decreases in c. Suppose market A is relatively semi-inelastic, that is $p_A/\varepsilon_A - p_B/\varepsilon_B > 0$. Default everyone into market A by setting $\alpha = 1$ and increase c by a little bit. By (9), dq_A is positive, and so consumers are shifted into market A. By (8), total seller surplus goes up.

The next lemma is the intermediate result that helps understand the outcome of the market equilibrium. It shows that the decentralized economy makes the same search decisions as the social planner who treats prices as given and maximizes consumer surplus with respect to the buyers' search decisions. In other words, starting from equilibrium outcome (x_B, x_A, p_B, p_A), the social planner cannot improve the net consumer surplus by choosing other search thresholds (x_B, x_A) while taking the prices, c and α fixed.

Lemma 2. *For any $c \geq 0$ and α, the equilibrium values of (x_B, x_A) are equal to those the social planner would choose to maximize the net consumer surplus, taking the equilibrium prices as given.*

The lemma follows from the fact that the consumers do not internalize the effect of their search decisions on prices because they take prices as given.

Now suppose the platform's objective is maximizing the net consumer surplus. For concreteness, let $p_A/\varepsilon_A > p_B/\varepsilon_B$, that is market A is more expensive and/or inelastic while market B is cheap and/or price elastic. By Theorem 2, the first-best consumer optimal allocation has more consumers in market B relative to the market equilibrium with $c = 0$. Even though we know from Theorem 6 that the first-best consumer surplus is not achieavable in a market equilibrium, we could hope to increase NCS using the policy described in Theorem 9 for increasing the sellers' profits. Namely, steer consumers into market B by defaulting them into good B and raise the switching cost. This policy increases gross consumer surplus because price in market B barely changes while the price in market A drops. If the resulting price differential is large enough, it compensates the losses from higher search costs. It turns out, however, that the increase in GCS is always smaller that the increase in the search costs, and so raising c above zero can only decrease NCS. This is the second main result of the paper.

Theorem 10 (Alignment theorem)**.** *A platform that wants to maximize net consumer surplus sets $c = 0$.*

This result implies that maximizing either welfare or the net consumer surplus require the same policy of the frictionless search.[4]

The next result is in line with the existing results in the intermediation literature and shows that generically the platform can increase both net consumer surplus, sellers' profits and welfare by diverting consumers. The difference from Hagiu and Jullien (2011) is that the platform does not observe the consumer type and set the default product that applies to everyone. Nevertheless, the diversion is effective whenever $c > 0$.

Theorem 11 (Anonymous diversion)**.** *Suppose $c > 0$. Then:*

- *Welfare is maximized with deterministic default, i.e. the optimal α is either 0 or 1.*

[4] Remember that when $c = 0$, changing α has no bite.

- *The joint sellers' profits can be increased by increasing the default probability of the good with higher* p_j/ε_j.

- *The net consumer surplus increases in* α *if* $q_A - q_B > K\,(p_A/\varepsilon_A - p_B/\varepsilon_B)$ *for some* $K \in (0, 2/\min_x\{v_A{}'(x) - v_B{}'(x)\})$.

6 Implications for Two-sided Search Intermediaries

In this section we discuss the implication for design of two-sided search intermediaries with respect to manipulating the consumer search costs and setting the default (recommended) products. So far, we have been thinking about the platform abstractly, as an institution with different potential objectives. Now we assume the platform charges fees and has a profit function. What would it optimally do?

First, assume that the platform takes a share τ of every seller's revenue, and the platform's objective is to maximize its revenue. The next result shows that in this situation the platform also has incentives to maintain positive search costs with the choice of the default product similar to the logic when maximizing the sellers' profits.

Theorem 12. *Suppose the platform takes a fixed cut* τ *from sellers' revenue. A platform that maximizes its revenue should set* $c > 0$. *A good with higher* $p_j(1 + \varepsilon_j^{-1})$ *should be set as the default, that is the more expensive good with less elastic supply.*

The default good is the good with lower semi-elasticity of supply and higher price, but compared to the profit-maximizing platform, there is more weight on price (compare with Theorem 9).

Now consider the case of vertical integration of the intermediary with one of the product markets, e.g. Amazon selling Amazon products on its marketplace, Google Flight Search on Google. Suppose the platform earns a profit share of τ in market A, but takes all the profit in market B. What does the platform optimally do now? The next result shows that the answer is a modification of Theorem 9, so that consumers are defaulted into the less semi-elastic market but favoring market B in weighting.

Theorem 13. *Suppose the platform is vertically integrated with market* B *and takes share* τ *of profit in market* A. *The platform optimally sets* $c > 0$. *The default is good* A *if* $\tau p_A/\varepsilon_A > p_B/\varepsilon_B$ *and* B*, otherwise.*

If supply elasticities are similar in both markets, and the platform's fee τ is small, then the platform has strong incentives to give prominence to its own product B by setting it as the default and maintaining the positive search cost.

7 Conclusion

Recent years have seen substantial investments by platforms in reducing search costs. Yet, there has been relatively little work on understanding what the implications of decreased search costs are for the parties involved. In this paper, we have provided positive results to shed some light into the effects of search on platform on welfare and revenue, which may help explain and guide platform policy related to search investments. Even though search frictinos lead to lower welfare generated on the platform, we showed that on the margin of frictionless search, the platform can generically increase sellers' revenue. If the platform takes a cut of the sellers' revenue, it has strong incentives to maintain positive search costs. On the other hand, if the platform favors consumers, then the platform would want to invest in decreasing search.

It is generally recognized that online markets attract more buyers than offline markets by allowing the consumers to find very niche and unique products (e.g. Ellison and Ellison (2009)). As such, easier search raises demand on the platform. At the same time, it introduces competition between providers because it is easy for a buyer to switch between the providers once on the platform. The analysis of this situation requires a richer model with oligopolistic market structure and endogenous participation. We hope to develop this extension in the future work.

Appendix

A Proofs

Proof. [Proof of Lemma 1]

Producer surplus—depends only on prices and quantities. Therefore, it is impossible to improve over a cutoff allocation with given (q_A, q_B). If $q_A + q_B < 1$, then a better allocation is to increase q_A until the full coverage. This works because the Full Coverage assumption guarantees all buyers will buy, and so prices follow quantities by $p_j = S_j^{-1}(q_j)$.

Consumer surplus. Suppose $q_A + q_B = 1$ but the allocation is fuzzy. The cutoff allocation with the same quantities would put all $x < q_B$ into B and all $x > q_B$ in to A . Since the allocation is fuzzy there is $x_2 > q_B$ who are in market B and $x_1 < q_B$ who are in market A. Take mass dx around x_2 and mass dx around x_1 and swap them. The gain in consumer surplus is

$$dx\,(u_A(x_2) - u_B(x_2) + u_B(x_1) - u_A(x_1)) = dx\,(u_A(x_2) - u_A(x_1) + u_B(x_1) - u_B(x_2)) \geq 0,$$

because u_A is increasing and u_B is decreasing. Suppose $q_A + q_B < 1$. Then you can arbitrary allocate the leftover consumers to markets because $u_j(x) \geq 0$ for $j = A, B$.

Welfare—is equal to the sum of producer and consumer suprlus. Since consumer surplus can be improved, and producer surplus is neutral to shuffling, the total welfare is maximized at a cutoff allocation.

Sellers' revenue. Similar argument as for the producer surplus.

□

Proof. [Proof of Theorem 1]

By Lemma 1, the optimal allocation is characterized by a cutoff x^* so that consumers $x < x^*$ are in market B and $x > x^*$ are in market A. With a cutoff allocation, the demand in markets A and B are

$$D_A(p) = \begin{cases} 1 - v_A^{-1}(p), & p_A > v_A(x^*) \\ 1 - x^*, & p_A < v_A(x^*) \end{cases} \tag{10}$$

$$D_B(p) = \begin{cases} 1 - v_B^{-1}(p), & p_B > v_B(x^*) \\ x^*, & p_B < v_B(x^*) \end{cases}$$

The total welfare is

$$W = \int_0^{p_A} S_A(t)dt + \int_{p_A}^{\bar{v}} D_A(t)dt + \int_0^{p_B} S_B(t)dt + \int_{p_B}^{\bar{v}} D_B(t)dt.$$

Differentiate W with respect to x^*. At competitive prices, $S_j(p_j) = D_j(p_j)$ for $j = A, B$, and so:

$$\frac{dW}{dx^*} = -(v_A(x^*) - p_A) + v_B(x^*) - p_B.$$

At the optimum $dW/dx^* = 0$, which proves the theorem.

□

Proof. [Proof of Theorem 2]

We will deal sequentially with consumer surplus, producer surplus, and then revenue.

Consumer surplus. By Lemma 1, the optimal allocation is in cutoff form so that consumers $x < x^*$ are in market B and $x > x^*$ are in market A. The consumer surplus is

$$CS = \int_{p_A}^{\bar{v}} D_A(t)dt + \int_{p_B}^{\bar{v}} D_B(t)dt.$$

Using (10),

$$\frac{dCS}{dx^*} = -(v_A(x^*) - p_A) - \frac{dp_A}{dx^*}q_A + v_B(x^*) - p_B - \frac{dp_B}{dx^*}q_B$$

Since the allocation is in cutoff form, $q_A = 1 - x^*$ and $q_B = x^*$.

$$\frac{dCS}{dx^*} = -(v_A(x^*) - p_A) + \frac{dp_A}{dq_A}q_A + v_B(x^*) - p_B - \frac{dp_B}{dq_B}q_B$$

In price equilibrium, $S_j(p_j) = q_j$, and so

$$\begin{aligned}\frac{dCS}{dx^*} &= -(v_A(x^*) - p_A) + \frac{p_A}{\varepsilon_A} + v_B(x^*) - p_B - \frac{p_B}{\varepsilon_B} \\ &= v_B(x^*) - p_B - (v_A(x^*) - p_A) + \frac{p_A}{\varepsilon_A} - \frac{p_B}{\varepsilon_B}. \end{aligned} \tag{11}$$

Evaluated at the welfare-optimal allocation x^{FBW},

$$\left.\frac{dCS}{dx^*}\right|_{x^*=x^{FBW}} = \frac{p_A}{\varepsilon_A} - \frac{p_B}{\varepsilon_B}.$$

Therefore, the consumer-optimal cutoff $x^{FBC} > x^{FBW}$ if and only if $\frac{p_A}{\varepsilon_A} - \frac{p_B}{\varepsilon_B} > 0$.

Sellers' profits. Again, by Lemma 1, the optimal allocation is in cutoff form. The sellers' profits is:

$$PS = \int_0^{p_A} S_A(t)dt + \int_0^{p_B} S_B(t)dt.$$

$$\frac{dPS}{dx^*} = \frac{dp_A}{dx^*}q_A + \frac{dp_B}{dx^*}q_B = -\frac{dp_A}{dq_A}q_A + \frac{dp_B}{dq_B}q_B = -\frac{p_A}{\varepsilon_A} + \frac{p_B}{\varepsilon_B}. \tag{12}$$

Therefore, the seller-optimal cutoff $x^{FBP} < x^{FBW}$ if and only if $\frac{p_A}{\varepsilon_A} - \frac{p_B}{\varepsilon_B} > 0$.

Sellers' revenue. Similar to what we did above,

$$R = p_A q_A + p_B q_B.$$

$$\frac{dR}{dx^*} = -p_A\left(\frac{1}{\varepsilon_A} + 1\right) + p_B\left(\frac{1}{\varepsilon_B} + 1\right).$$

Therefore, generically, $x^{FBR} \neq x^{FBW}$. Also,

$$\frac{dR}{dx^*} = \frac{dPS}{dx^*} + p_B - p_A,$$

and so $x^{FBR} \neq x^{FBP}$.

□

Proof of Theorem 3. We prove the theorem in two parts. First, we show existence with Brouwer's fixed point theorem. We can characterize equilibria by ordered threshold pairs $(x_b, x_a) \in [0,1]^2$ — such that $x < x_b$ search into B, $x > x_a$ search into A and the rest do not search — and let $T : [0,1]^2 \to [0,1]^2$ be a mapping from a pair of buyer thresholds to a new pair that describes those buyers indifferent between searching (either into A or B) and not searching. Prices p_A and p_B as solutions of (2) are continuous functions of (x_B, x_A). Thus mapping T is continuous. The set $[0,1]^2$ is compact, so Brouwer's theorem ensures the existence of a fixed point, which is an equilibrium.

Uniqueness is more involved. First we introduce a bit of notation. Let $\mathbf{x} = (x_B, x_A)$, and define $s(x; \mathbf{x}) = u^A(x) - u^B(x)$ be the difference between A and B market utilities for a type x buyer — since this depends on the behaviors of other buyers, we parameterize this by the thresholds. Then let $\mathbf{S}(\mathbf{x}) \equiv (s(x_B; \mathbf{x}), s(x_A; \mathbf{x}))$ give the market differences to the threshold types.

Equilibria are characterized by

$$\begin{cases} u^A(x_B) - u^B(x_B) = -c \\ u^A(x_A) - u^B(x_A) = c \end{cases}$$

which we rewrite in a concise form as

$$\mathbf{S}(\mathbf{x}) = (-c, c) = \mathbf{c}. \tag{13}$$

Below we show that the principal minors of Jacobian matrix of $\mathbf{S}$ are positive[5] everywhere. Then by Gale and Nikaido (1965), the mapping is one-to-one and hence $\mathbf{S}(\mathbf{x}) = \mathbf{c}$ has a unique solution.

The Jacobian matrix is

$$J = \begin{bmatrix} \frac{ds(x_B)}{dx_B} & \frac{ds(x_B)}{dx_A} \\ \frac{ds(x_A)}{dx_B} & \frac{ds(x_A)}{dx_A} \end{bmatrix}. \tag{14}$$

There are four cases of (x_B, x_A) we need to deal with separately:

I. $v_A^{-1}(p_A) < x_B \le x_A < v_B^{-1}(p_B)$,

II. $v_A^{-1}(p_A) < x_B < v_B^{-1}(p_B) < x_A$,

III. $x_B < v_A^{-1}(p_A) < x_A < v_B^{-1}(p_B)$,

IV. $x_B < v_A^{-1}(p_A) < v_B^{-1}(p_B) < x_A$.

Full Coverage assumption implies that $v_A^{-1}(p_A) < v_B^{-1}(p_B)$, and so there are only four cases described above.

[5] We use "positive" and "nonnegative" language in the paper.

Case I. In this case, $p_A < v_A(x_B)$ and $p_B < v_B(x_A)$. We can find how prices change with x_B and x_A:

$$\frac{dp_A}{dx_A} = -\frac{1-\alpha}{S'_A(p_A)} \qquad \frac{dp_A}{dx_B} = -\frac{\alpha}{S'_A(p_A)}$$
$$\frac{dp_B}{dx_B} = \frac{\alpha}{S'_B(p_B)} \qquad \frac{dp_B}{dx_A} = \frac{1-\alpha}{S'_B(p_B)}$$

We have

$$\frac{du_A(x_A)}{dx_A} = v'_A(x_A) + \frac{1-\alpha}{S'_A(p_A)}$$
$$\frac{du_A(x_A)}{dx_B} = \frac{\alpha}{S'_A(p_A)}$$
$$\frac{du_B(x_A)}{dx_A} = v'_B(x_A) - \frac{1-\alpha}{S'_B(p_B)}$$
$$\frac{du_B(x_A)}{dx_B} = -\frac{\alpha}{S'_B(p_B)}$$

And so,

$$\frac{ds(x_A)}{dx_A} = \underbrace{v'_A(x_A) - v'_B(x_A)}_{=:X_1} + \underbrace{\frac{1-\alpha}{S'_A(p_A)} + \frac{1-\alpha}{S'_B(p_B)}}_{=:X_3} \tag{15}$$
$$\frac{ds(x_A)}{dx_B} = \frac{\alpha}{S'_A(p_A)} + \frac{\alpha}{S'_B(p_B)} =: X_4 \tag{16}$$

Similarly,

$$\frac{du_A(x_B)}{dx_A} = \frac{1-\alpha}{S'_A(p_A)}$$
$$\frac{du_A(x_B)}{dx_B} = v'_A(x_B) + \frac{\alpha}{S'_A(p_A)}$$
$$\frac{du_A(x_B)}{dx_A} = -\frac{1-\alpha}{S'_B(p_B)}$$
$$\frac{du_B(x_B)}{dx_B} = v'_B(x_B) - \frac{\alpha}{S'_B(p_B)}$$

And so,

$$\frac{ds(x_B)}{dx_B} = \underbrace{v'_A(x_B) - v'_B(x_B)}_{=:X_2} + \underbrace{\frac{\alpha}{S'_A(p_A)} + \frac{\alpha}{S'_B(p_B)}}_{=X_4} \tag{17}$$
$$\frac{ds(x_B)}{dx_A} = \frac{1-\alpha}{S'_A(p_A)} + \frac{1-\alpha}{S'_B(p_B)} = X_3 \tag{18}$$

$$J = \begin{pmatrix} X_2 + X_4 & X_3 \\ X_4 & X_1 + X_3 \end{pmatrix}$$

The Jacobian is equal to

$$\begin{aligned} \det J &= \frac{ds(x_B)}{dx_B}\frac{ds(x_A)}{dx_A} - \frac{ds(x_B)}{dx_A}\frac{ds(x_A)}{dx_B} \\ &= (X_2 + X_4)(X_1 + X_3) - X_3 X_4 = X_1 X_2 + X_1 X_4 + X_2 X_3. \end{aligned}$$

Since v_A is increasing, v_B is decreasing and the supply curves are increasing, we have that $X_j > 0$ for all $j = 1, 2, 3, 4$. Therefore,

$$\det J > 0.$$

Additionally, all diagonal elements of Jacobian matrix are positive, $\frac{ds(x_B)}{dx_B} > 0$ and $\frac{ds(x_A)}{dx_A} > 0$. Therefore, Gale and Nikaido (1965) applies and the solution to (13) is unique.

The other cases are done in a similar fashion, and we provide all the necessary calculations in the rest of the proof.

Case II. In this case, $p_A < v_A(x_B)$ and $v_B(x_B) > p_B > v_B(x_A)$.

$$\begin{aligned} \frac{dp_A}{dx_A} &= -\frac{1-\alpha}{S'_A(p_A)} & \frac{dp_A}{dx_B} &= -\frac{\alpha}{S'_A(p_A)} \\ \frac{dp_B}{dx_B} &= \frac{\alpha}{S'_B(p_B) - \frac{1-\alpha}{v_B{}'(v_B{}^{-1}(p_B))}} & \frac{dp_B}{dx_A} &= 0 \end{aligned}$$

We find:

$$\begin{aligned} \frac{ds(x_A)}{dx_A} &= v'_A(x_A) + \frac{1-\alpha}{S'_A(p_A)} > 0 \\ \frac{ds(x_A)}{dx_B} &= \frac{\alpha}{S'_A(p_A)} \end{aligned}$$

$$\begin{aligned} \frac{ds(x_B)}{dx_B} &= v'_A(x_B) - v'_B(x_B) + \frac{\alpha}{S'_A(p_A)} + \frac{\alpha}{S'_B(p_B) - \frac{1-\alpha}{v_B{}'(v_B{}^{-1}(p_B))}} > 0 \\ \frac{ds(x_B)}{dx_A} &= \frac{1-\alpha}{S'_A(p_A)} \end{aligned}$$

It is easy to check that $\det J > 0$.

Case III. In this case, $v_A(x_A) > p_A > v_A(x_B)$ and $p_B < v_B(x_A)$.

$$\frac{dp_A}{dx_A} = -\frac{1-\alpha}{S'_A(p_A) + \frac{\alpha}{v_A{}'(v_A{}^{-1}(p_A))}} \qquad \frac{dp_A}{dx_B} = 0$$
$$\frac{dp_B}{dx_B} = \frac{\alpha}{S'_B(p_B)} \qquad \frac{dp_B}{dx_A} = \frac{1-\alpha}{S'_B(p_B)}$$

We find:

$$\frac{ds(x_A)}{dx_A} = v'_A(x_A) - v_B{}'(x_A) + \frac{1-\alpha}{S'_A(p_A) + \frac{\alpha}{v_A{}'(v_A{}^{-1}(p_A))}} + \frac{1-\alpha}{S'_B(p_B)} > 0$$
$$\frac{ds(x_A)}{dx_B} = \frac{\alpha}{S'_B(p_B)}$$

$$\frac{ds(x_B)}{dx_B} = -v'_B(x_B) + \frac{\alpha}{S'_B(p_B)} > 0$$
$$\frac{ds(x_B)}{dx_A} = \frac{1-\alpha}{S'_B(p_B)}$$

It is easy to check that $\det J > 0$.

Case IV. In this case, $v_A(x_A) > p_A > v_A(x_B)$ and $v_B(x_B) > p_B > v_B(x_A)$.

$$\frac{dp_A}{dx_A} = -\frac{1-\alpha}{S'_A(p_A) + \frac{\alpha}{v_A{}'(v_A{}^{-1}(p_A))}} \qquad \frac{dp_A}{dx_B} = 0$$
$$\frac{dp_B}{dx_B} = \frac{\alpha}{S'_B(p_B) - \frac{1-\alpha}{v_B{}'(v_B{}^{-1}(p_B))}} \qquad \frac{dp_B}{dx_A} = 0$$

We find:

$$\frac{ds(x_A)}{dx_A} = v'_A(x_A) + \frac{1-\alpha}{S'_A(p_A) + \frac{\alpha}{v_A{}'(v_A{}^{-1}(p_A))}} > 0$$
$$\frac{ds(x_A)}{dx_B} = 0$$

$$\frac{ds(x_B)}{dx_B} = -v'_B(x_B) + \frac{\alpha}{S'_B(p_B) - \frac{1-\alpha}{v_B{}'(v_B{}^{-1}(p_B))}} > 0$$
$$\frac{ds(x_B)}{dx_A} = 0$$

It is easy to see that $\det J > 0$.

□

Proof of Theorem 4. We need to sign the terms of the inverse of the Jacobian matrix. Applying the implicit function theorem to $\mathbf{S}(\mathbf{x}) = \mathbf{c}$ and substituting in the known signs of the inverse Jacobian matrix gives monotonicity.

Positive principal minors of Jacobian (14) imply that the inverse of the Jacobian matrix is signed as follows (Cramer's rule):

$$\operatorname{sgn}(J^{-1}) = \begin{pmatrix} + & - \\ - & + \end{pmatrix}$$

Therefore,

$$\begin{pmatrix} \frac{dx_B}{dc} \\ \frac{dx_A}{dc} \end{pmatrix} = J^{-1} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

and so,

$$\frac{dx_B}{dc} < 0 \qquad \frac{dx_A}{dc} > 0.$$

□

Proof of Theorem 5. As shown in (13),

$$\mathbf{S}(\mathbf{x}; \alpha) = (-c, c). \tag{19}$$

Our goal is to find $dx_A/d\alpha$ and $dx_B/d\alpha$. To this end, differentiate (19) with respect to α:

$$J \cdot \begin{pmatrix} \frac{dx_B}{d\alpha} \\ \frac{dx_A}{d\alpha} \end{pmatrix} + \frac{\partial \mathbf{S}}{\partial \alpha} = 0, \tag{20}$$

where J is given in (14). When all buyers purchase a product, $s(x) = u_A(x) - u_B(x) = v_A(x) - p_A - (v_B(x) - p_B)$. The only part that depends directly on α is the prices. Again, When all buyers purchase a product,

$$S_A(p_A) = 1 - x_A + \alpha(x_A - x_B)$$
$$S_B(p_B) = x_B + (1 - \alpha)(x_A - x_B)$$

From here we find

$$\frac{dp_A}{d\alpha} = \frac{x_A - x_B}{S'_A(p_A)}$$
$$\frac{dp_B}{d\alpha} = -\frac{x_A - x_B}{S'_B(p_B)}$$

Therefore,

$$\frac{\partial \mathbf{S}}{\partial \alpha} = \begin{pmatrix} \frac{\partial (p_B - p_A)}{\partial \alpha} \\ \frac{\partial (p_B - p_A)}{\partial \alpha} \end{pmatrix} = -(x_A - x_B) \left(\frac{1}{S'_A(p_A)} + \frac{1}{S'_B(p_B)} \right) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Plugging back to (20),

$$\begin{aligned} \begin{pmatrix} \frac{dx_B}{d\alpha} \\ \frac{dx_A}{d\alpha} \end{pmatrix} &= J^{-1} \frac{\partial \mathbf{S}}{\partial \alpha} \\ &= \frac{1}{\det J} \begin{pmatrix} X_1 + X_3 & -X_3 \\ -X_4 & X_2 + X_4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} (x_A - x_B) \left(\frac{1}{S'_A(p_A)} + \frac{1}{S'_B(p_B)} \right), \end{aligned}$$

where $X_j \geq 0$, $j = 1, 2, 3, 4$ are defined in (15) and (17).

$$\begin{pmatrix} \frac{dx_B}{d\alpha} \\ \frac{dx_A}{d\alpha} \end{pmatrix} = \frac{1}{\det J} \begin{pmatrix} X_1 \\ X_4 \end{pmatrix} (x_A - x_B) \left(\frac{1}{S'_A(p_A)} + \frac{1}{S'_B(p_B)} \right) \tag{21}$$

$$= \frac{1}{\det J} \begin{pmatrix} v'_A(x_A) - v'_B(x_A) \\ v'_A(x_B) - v'_B(x_B) \end{pmatrix} (x_A - x_B) \left(\frac{1}{S'_A(p_A)} + \frac{1}{S'_B(p_B)} \right). \tag{22}$$

Since v_A is increasing and v_B is decreasing,

$$\frac{dx_B}{d\alpha} \geq 0$$
$$\frac{dx_A}{d\alpha} \geq 0$$

□

Proof. [Proof of Lemma 2] Consumer surplus is the surplus obtained in the trade phase less the search costs.

$$CS = \int_{x_A}^{1} u_A(x)dx + \int_{x_B}^{x_A} (\alpha u_A(x) + (1-\alpha)u_B(x))dx + \int_{0}^{x_B} u_B(x)dx - (1-\alpha)(1-x_A)c - \alpha x_B c.$$

Since prices are fixed, u_A and u_B do not change in response to changes in x_A and x_B. If x_A and x_B are chosen optimally to maximize the consumer surplus, then we have:

$$\frac{dCS}{dx_A} = -(1-\alpha)u_A(x_A) + (1-\alpha)c + (1-\alpha)u_B(x_A) = 0 \tag{23}$$

$$\frac{dCS}{dx_B} = -\alpha u_A(x_B) - (1-\alpha)u_B(x_B) + u_B(x_B) - \alpha c = 0 \tag{24}$$

These two equations coincide with the equilibrium conditions (6). □

Proof. [Proof of Theorem 7] By Lemma 2, (x_B, x_A) maximize the consumer surplus holding the prices and c and α fixed. Also, since prices are fixed, u_A and u_B do not change in response to changes in c.By the envelope theorem,

$$\frac{dCS}{dc} = -(1-\alpha)(1-x_A) - \alpha x_B < 0.$$

□

Proof. [Proof of Theorem 8]

$$R = q_A p_A + q_B p_B$$

By the Full Coverage assumption, when c is close to zero, all consumers still purchase a product. We have that $q_A = 1 - x_A + \alpha(x_A - x_B)$ and $q_B = x_B + (1-\alpha)(x_A - x_B)$.

$$\frac{dq_A}{dc} = -(1-\alpha)\frac{dx_A}{dc} - \alpha\frac{dx_B}{dc}$$
$$\frac{dq_B}{dc} = (1-\alpha)\frac{dx_A}{dc} + \alpha\frac{dx_B}{dc}$$

Therefore,

$$\frac{dR}{dc} = \frac{dx_A}{dc}(1-\alpha)(p_B - pA) + \frac{dx_B}{dc}\alpha(p_B - p_A)$$
$$= (p_B - p_A)(\frac{dx_A}{dc}(1-\alpha) + \alpha\frac{dx_B}{dc}).$$

As shown in Theorem 4, $dx_A/dc > 0$ and $dx_B/dc < 0$. Also, in the case of fixed prices, dx_A/dc and dx_B/dc are independent of α. Let good B be more expensive, $p_B > p_A$. To maximize dR/dc, the intermediary should set $\alpha = 0$, that is to set good B as the default. Therefore, while all consumers still purchase a product, it is optimal to increase c. □

Proof. [Proof of Theorem 9]

In the text after the state of the theorem, we established that when all buyers purchase a product,

$$dPS = \left(\frac{p_A}{\varepsilon_A} - \frac{p_B}{\varepsilon_B}\right) dq_A. \tag{25}$$

The rest of the discussion in the text was informal, and here we provide a rigorous version.

In price equilibrium when all buyers purchase a product,

$$q_A = 1 - x_A + \alpha(x_A - x_B),$$

from where

$$\frac{dq_A}{dc} = -(1-\alpha)\frac{dx_A}{dc} - \alpha\frac{dx_B}{dc}.$$

Start with $c = 0$. We will show that there is α and a deviation to $c > 0$ that increases PS. As shown in Theorem 4, $dx_A/dc > 0$ and $dx_B/dc < 0$. If $\frac{p_B}{\varepsilon_B} - \frac{p_A}{\varepsilon_A} > 0$, then set $\alpha = 0$ and increase c by a little bit. By (25), the joint seller profit increases. Conversely, if $\frac{p_B}{\varepsilon_B} - \frac{p_A}{\varepsilon_A} < 0$, then set $\alpha = 1$ and increase c. Therefore, the default good should be the one with less semi-elastic supply.

Since dx_A/dc and dx_B/dc depend on α, the corner values of α need not be optimal, which implies that the optimal default can be stochastic. To show that the optimal default is deterministic, evaluate $dPS/d\alpha$ at corner values of α with $c > 0$. For concreteness, let $\frac{p_B}{\varepsilon_B} < \frac{p_A}{\varepsilon_A}$, which implies that defaulting everyone into A helps to increase the profits. Can we do better by lowering α?

$$\frac{dPS}{d\alpha} = \left(\frac{p_A}{\varepsilon_A} - \frac{p_B}{\varepsilon_B}\right)\frac{dq_A}{d\alpha}.$$

By Lemma 4, $dq_A/d\alpha > 0$ when $c > 0$, and so $dPS/d\alpha > 0$. Therefore, $\alpha = 1$ is the global maximum. Similarly one can find that when $\frac{p_B}{\varepsilon_B} > \frac{p_A}{\varepsilon_A}$, $dPS/d\alpha < 0$.

□

Proof. [Proof of Theorem 10] The net consumers surplus is the utility the consumers obtain from buying a product, less price, less search costs:

$$NCS = \int_{p_A}^{\bar{v}} D_A(t)dt - (1-\alpha)(1-x_A)c + \int_{p_B}^{\bar{v}} D_B(t)dt - \alpha x_B c.$$

The equilibrium thresholds (x_B, x_A) are as if the social planner maximized the consumer surplus holding the prices fixed (Lemma 2). Therefore differentiating NCS with respect to c we don't need to take the partial derivative with respect to (x_B, x_A). We have:

$$NCS = \int_{p_A}^{\bar{v}} D_A(t)dt - (1-\alpha)(1-x_A)c + \int_{p_B}^{\bar{v}} D_B(t)dt - \alpha x_B c.$$

The equilibrium thresholds (x_B, x_A) are as if the social planner maximized the consumer surplus holding the prices fixed (Lemma 2). Therefore differentiating NCS with respect to c we don't need to take the partial derivative with respect to (x_B, x_A). We have:

$$\begin{aligned}\frac{dNCS}{dc} &= -\frac{dp_A}{dc}q_A - \frac{dp_B}{dc}q_B - (1-\alpha)(1-x_A) - \alpha x_B \\ &= -\left(\frac{p_A}{\varepsilon_A} - \frac{p_B}{\varepsilon_B}\right) dq_A - (1-\alpha)(1-x_A) - \alpha x_B \\ &= \left(\frac{p_A}{\varepsilon_A} - \frac{p_B}{\varepsilon_B}\right)\left((1-\alpha)\frac{dx_A}{dc} + \alpha\frac{dx_B}{dc}\right) - (1-\alpha)(1-x_A) - \alpha x_B \qquad (26)\end{aligned}$$

When all consumers purchase a product, we have

$$\begin{aligned}\frac{p_A}{\varepsilon_A} &= \frac{q_A}{S'_A} = \frac{1 - x_A + \alpha(x_A - x_B)}{S'_A} \\ \frac{p_B}{\varepsilon_B} &= \frac{q_B}{S'_B} = \frac{x_B + (1-\alpha)(x_A - x_B)}{S'_B}\end{aligned}$$

Using (13) and (14), obtain that

$$\begin{aligned}\begin{pmatrix} dx_B/dc \\ dx_A/dc \end{pmatrix} &= J^{-1}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \frac{1}{\det J}\begin{pmatrix} X_1 + X_3 & -X_3 \\ -X_4 & X_2 + X_4 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \\ &= \frac{1}{\det J}\begin{pmatrix} -X_1 - 2X_3 \\ X_2 + 2X_4 \end{pmatrix}.\end{aligned}$$

Adopt notation $h(x) = v_A{}'(x) - v_B{}'(x)$. Then

$$\begin{aligned}X_1 &= h(x_A) \\ X_2 &= h(x_B) \\ X_3 &= (1-\alpha)(S'^{-1}_A + S'^{-1}_B) \\ X_4 &= \alpha(S'^{-1}_A + S'^{-1}_B)\end{aligned}$$

Further,

$$(1-\alpha)\frac{dx_A}{dc} + \alpha\frac{dx_B}{dc} = \frac{(1-\alpha)h(x_B) - \alpha h(x_A)}{\det J} \qquad (27)$$

$$\begin{aligned}\det J &= X_1X_2 + X_1X_4 + X_2X_3 = \\ &= h(x_A)h(x_B) + h(x_A)\alpha(S'^{-1}_A + S'^{-1}_B) + h(x_B)(1-\alpha)(S'^{-1}_A + S'^{-1}_B) \\ &= h(x_A)h(x_B) + (h(x_A)\alpha + h(x_B)(1-\alpha))(S'^{-1}_A + S'^{-1}_B) \qquad (28)\end{aligned}$$

Plug back all expression into (26). We have

$$\frac{dNCS}{dc} = \left(\frac{1 - x_A + \alpha(x_A - x_B)}{S'_A} - \frac{x_B + (1-\alpha)(x_A - x_B)}{S'_B}\right)\frac{(1-\alpha)h(x_B) - \alpha h(x_A)}{\det J}$$
$$- (1-\alpha)(1 - x_A) - \alpha x_B.$$

Look at the numerator only, and collect terms by $S_A'^{-1}$ and $S_B'^{-1}$. The multiplier on $S_A'^{-1}$ in the numerator of $\frac{dNCS}{dc}$ is:

$$\begin{gathered}
[1 - x_A + \alpha(x_A - x_B)]\,[(1-\alpha)h(x_B) - \alpha h(x_A)] \\
- [(1-\alpha)(1-x_A) + \alpha x_B]\,[h(x_A)\alpha + h(x_B)(1-\alpha)] \\
= h(x_A)\alpha\,[-(1 - x_A + \alpha(x_A - x_B)) - [(1-\alpha)(1-x_A) + \alpha x_B]] \\
+ h(x_B)(1-\alpha)\,[1 - x_A + \alpha(x_A - x_B) - [(1-\alpha)(1-x_A) + \alpha x_B]] \\
= h(x_A)\alpha\,[-(1 - x_A + \alpha x_A) - [(1-\alpha)(1-x_A)]] \\
+ h(x_B)(1-\alpha)\,[1 + \alpha(-x_B) - [(1-\alpha) + \alpha x_B]] \\
= h(x_A)\alpha\,[-2 + \alpha + (2-2\alpha)x_A] + h(x_B)(1-\alpha)\,[\alpha - 2\alpha x_B] < 0.
\end{gathered}$$

The multiplier on $S_B'^{-1}$ in the numerator of $\frac{dNCS}{dc}$ is:

$$\begin{gathered}
-[x_B + (1-\alpha)(x_A - x_B)]\,[(1-\alpha)h(x_B) - \alpha h(x_A)] \\
- [(1-\alpha)(1-x_A) + \alpha x_B]\,[h(x_A)\alpha + h(x_B)(1-\alpha)] \\
= h(x_A)\alpha\,[x_B + (1-\alpha)(x_A - x_B) - [(1-\alpha)(1-x_A) + \alpha x_B]] \\
+ h(x_B)(1-\alpha)\,[-[x_B + (1-\alpha)(x_A - x_B)] - [(1-\alpha)(1-x_A) + \alpha x_B]] \\
= h(x_A)\alpha\,[-1 + \alpha + (2-2\alpha)x_A] + h(x_B)(1-\alpha)\,[-1 + \alpha - 2\alpha x_B] < 0.
\end{gathered}$$

The remaining free term in the numerator of $\frac{dNCS}{dc}$ is:

$$-h(x_A)h(x_B)((1-\alpha)x_A + \alpha x_B) < 0.$$

Therefore,

$$\frac{dNCS}{dc} < 0.$$

□

Lemma 3. *Let $R = R_A + R_B$ be joint seller revenue, where $R_j = p_j q_j$. Then for c when all consumers buy a product,*

$$\frac{dR}{dc} = \left(p_B\frac{1+\varepsilon_B^S}{\varepsilon_B^S} - p_A\frac{1+\varepsilon_A^S}{\varepsilon_A^S}\right)\left((1-\alpha)\frac{dx_A}{dc} + \alpha\frac{dx_B}{dc}\right) \tag{29}$$

Proof.

$$R_A = p_A S_A(p_A)$$

$$\begin{aligned}
\frac{dR_A}{dc} &= \frac{dp_A}{dc}q_A + S'_A(p_A)\frac{dp_A}{dc}p_A \\
&= \frac{dp_A}{dc}q_A(1+\varepsilon_A^S).
\end{aligned}$$

Plug in the expression for $\frac{dp_A}{dc}$ from (**??**), and obtain the result. □

Lemma 4. *Suppose $c > 0$. Then*

$$dq_A/d\alpha = \frac{(v_A{}'(x_B) - v_B{}'(x_B))(v_A{}'(x_A) - v_B{}'(x_A))}{\det J}(x_A - x_B) > 0.$$

Proof. When all buyers purchase a product,

$$q_A = 1 - x_A + \alpha(x_A - x_B).$$

$$\frac{dq_A}{d\alpha} = -(1-\alpha)\frac{dx_A}{d\alpha} - \alpha\frac{dx_B}{d\alpha} + x_A - x_B.$$

Using (22),

$$\frac{dq_A}{d\alpha} = -\frac{(x_A - x_B)(S_A'^{-1} + S_B'^{-1})}{\det J}((1-\alpha)h(x_B) + \alpha h(x_A)) + x_A - x_B,$$

where $h(x) = v_A{}'(x) - v_B{}'(x)$. Plug in the expression for $\det J$ from (28):

$$\begin{aligned}\frac{\det J}{x_A - x_B}\frac{dq_A}{d\alpha} &= -(S_A'^{-1} + S_B'^{-1})((1-\alpha)h(x_B) + \alpha h(x_A)) + h(x_A)h(x_B) \\ &\quad + (h(x_A)\alpha + h(x_B)(1-\alpha))(S_A'^{-1} + S_B'^{-1}) \\ &= h(x_A)h(x_B).\end{aligned}$$

When $c > 0$, we have that $x_A > x_B$.

□

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